Necessitarianism Lost: Defusing a new argument for modal collapse

Abstract. Hall 2021 gives a rigorous proof of the so-called modal collapse argument against the principle of sufficient reason (PSR): If PSR is true, then all propositions are necessary propositions; but not all propositions are necessary; so PSR is false. I prove that, when we supplement the theory in which he derives the argument, `TSR`, with a principle (namely that conjuncts are a sufficient reason for their conjunction) which very plausibly must accompany any formalization of the PSR, the resulting theory $\texttt{TSR}^{+}$ is inconsistent. I further prove that, if we supplement $\texttt{TSR}^{+}$ with a modified conjunction principle and a principle which rules out the possibility that a proposition p may, along with other propositions, be its own sufficient reason, the resulting theory $\texttt{TSR}^{**}$ is also inconsistent. Importantly, this result is reached *without* use of the PSR. I conclude that we should reject the assumption of *distributivity*, which holds that if p is the sufficient reason for some conjunction $\bigwedge qq$, then p is the sufficient reason for every conjunct q. But without distributivity, the modal collapse argument fails.

Introduction

There is a famous argument against the principle of sufficient reason (=PSR), originated by Jonathan Bennett and Peter van Inwagen (in Bennett 1984, 114–118 and van Inwagen 1986, 202–204, respectively) which alleges that the principle leads to necessitarianism, the doctrine that there are no contingent truths. Here is Sam Levey's rendering of it:

> Let C be the conjunction of all contingent truths. Then C itself is a contingent truth, for no necessary truth can have a contingent truth as a conjunct. By PSR, there is an explanatory ground G that is a sufficient reason for C. G entails C and explains C. Is

> G itself a contingent truth? If so, then G is in C. But then in explaining C, G would also explain itself, and no contingent truth can explain itself. If G is not a contingent truth but a necessary truth, then because G entails C, it follows that C is a necessary truth, contrary to hypothesis. So, given PSR, there can be no conjunction C of all contingent truths. If there is no conjunction C of all contingent truths, then it must be that there are no contingent truths. Therefore, PSR entails that there are no contingent truths. (Levey 2016, 399–400)

Responses to this argument have been varied. Some (e.g. Tomaszewski 2016; Pruss 2006) find reasons to reject the argument. Somewhat fewer are those who embrace the necessitarian consequences (e.g. Della Rocca 2010). Levey's own response is to deny that there is a conjunction of all contingent truths, on the grounds that the concept of "contingent truth" is indefinitely extensible – that is, every collection of objects satisfying that concept allows one to identify a new object, distinct from the first ones, which also satisfies it.[1] (Levey 2016, 402)

But in philosophy's house there are many mansions, and none of their doors is forever closed. Recent work has resurrected versions of the van Inwagen-Bennett argument that purport to succeed in the face of some objections. McDaniel 2019 argues that when rendered in terms of grounding, extant responses to modal collapse arguments fail. And in a recent paper (Hall 2021), Geoffrey Hall has taken aim at Sam Levey's argument and shown rigorously that, given classical non-constructivist assumptions, the response fails. Necessitarianism is regained.

1. See Amijee 2020, §4 for a canvass and critical appraisal of the van Inwagen-Bennett argument and responses to it.

I do not dispute that Hall has offered a rigorous proof of necessitarianism from his principles. It is rigorous, and it is valid. Instead, I question its soundness. I will argue here that once one supplements Hall's premises with an extremely plausible principle – viz., that conjuncts help explain their conjunction – one can reach a contradiction. This allows us, I will argue, to diagnose what is wrong about Hall's original argument.

Here is the plan of the paper. In §1, I will present the language and the logic in which Hall formulates his argument, and state his main result. In §2, I will show that a natural extension of his principles and his logic is formally inconsistent, in the sense of entailing a formula and its negation. In §3, I show that on another natural modification of his principles, one can still reach a formal inconsistency. Finally, in §4, I attempt to diagnose the problem, and argue that one of his assumptions, that of *distributivity*, should be rejected.

Section 1. Hall's argument

Hall's argument uses a language with the following components (473–5):

- A countable collection of singular propositional variables p_1, p_2,. . .
- A countable collection of plural propositional variables, pp_1, pp_2,...
- Boolean operators for negation and conjunction, $\neg$ and $\wedge$
- A unary modal operator $\Box$ for metaphysical necessity
- A universal quantifier $\forall$ which binds singular propositional variables.
- A universal quantifier also written $\forall$ which binds plural propositional variables.
- A binary connective $<$ to formalize "φ is the sufficient reason for ψ", written as $\varphi < \psi$

- A binary connective $\prec$ to formalize "p is one of the propositions pp_i," written as $p \prec pp_i$
- A unary operator $\bigwedge$ which conjoins plural propositional variables to formalize "the conjunction of the propositions that pp_i," written as $\bigwedge pp_i$

Call this language $\mathcal{L}_{PSR}$. The usual definitions can be given for $\rightarrow$, $\vee$, $\leftrightarrow$, $\exists$, and $\Diamond$. Hall abbreviates $\varphi \wedge \Diamond\neg\varphi$ as $C\varphi$, encoding the idea of a contingent proposition: it both is true and can fail to be true. (Hall 2021, 473) Similarly, he abbreviates $\exists q(q < \varphi)$ as $E\varphi$. (473) The PSR is then:

(PSR) $\forall p(Cp \rightarrow Ep)$

The standard clauses about what constitutes a well-formed formula can be precisely formulated but don't concern us here. Hall then puts forward the following "background logic" consisting of axiom schemata and rules of inference (475):

(A1) Any substitution instance of a propositional tautology is an axiom.
(A2) $\forall x\varphi \rightarrow \varphi[x/t]$ where t is a term of appropriate sort free for x in φ.
(A3) $\forall x(\varphi \rightarrow \psi) \rightarrow (\forall x\varphi \rightarrow \forall x\psi)$
(A4) $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
(A5) $\exists qq\forall p[p \prec qq \leftrightarrow \varphi(p)]$ where neither p nor qq appear free in φ

(R1) φ, $\varphi \rightarrow \psi$ / ψ
(R2) $(\varphi \rightarrow \psi)$ / $(\varphi \rightarrow \forall x\psi)$ when x is not free in ψ
(R3) φ / $\Box\varphi$

The background logic, BL, is gotten by taking the closure of A1-A5 under R1-R3. We then write $\vdash_{\texttt{BL}} \varphi$ for $\varphi \in \texttt{BL}$.

The principles he lays out are (Hall 2021, 489):

(I) $\forall pp[\Box \bigwedge pp \to \forall p(p \prec pp \to \Box p)]$

(T) $\forall pp[\forall p(p \prec pp \to p) \to \bigwedge pp]$

(S) $\forall p \forall q(p < q \to \Box(p \to q))$

(Ir) $\forall p(p \not< p)$

(D) $\forall p \forall qq[(p < \bigwedge qq) \to \forall q(q \prec qq \to p < q)]$

(F) $\forall p \forall q[p < q \to (p \wedge q)]$

(PSR) $\forall p(Cp \to Ep)$

Call the set of all the above principles TSR. Hall then proves the following

THEOREM 1. *TSR* $\vdash_{BL} \forall p(p \to \Box p)$

In words, that all true propositions are necessarily true is a theorem of TSR under BL. (Appendix 1)

Section 2. TSR$^+$ is Inconsistent

I don't dispute Hall's proof. It is valid. The trouble is that by adding a plausible principle to TSR, the theory becomes inconsistent, where for a theory to be inconsistent in a derivation system in the present sense is for it to prove, in that derivation system and for some formula φ, φ and $\neg\varphi$. To get there, however, we need a little extra work.

First consider the extension of $\mathcal{L}_{PSR}$ gotten by letting multiple propositions taken together constitute a sufficient reason. We do this by adding a clause for a new connective $<^*$:

- An (n+1)-ary connective $<$ for every $n \in \mathbb{N}$ to formalize "φ_1, φ_2,...φ_n are together the minimal sufficient reason for ψ," written as $\varphi_1; \varphi_2 \ldots ; \varphi_n < \psi$.

The notion of a "minimal sufficient reason" is straightforward.

Definition 1 (Minimal sufficient reason). *pp taken together are the minimal sufficient reason for q* iff *(i)* p_1, p_2,...p_n *taken together are a sufficient reason for q, (ii)* p_1, p_2,...p_n - p_m *for* $1 \leq m \leq n$ *aren't, and (iii) there is no other plurality rr distinct from qq such that the rr fulfills (i) and (ii).*[2]

Intuitively a minimal sufficient reason is the "smallest" collection of propositions which constitute a sufficient reason for another one. We call the extension of $\mathcal{L}_{PSR}$ gotten by revising the clause for $<$ $\mathcal{L}^+_{PSR}$.

We may do the same thing with plural variables and constants: *pp* taken together are the minimal sufficient reason for q *iff* p_1, p_2,...p_n taken together are a sufficient reason for q p_1, p_2,...p_n - p_m for $1 \leq m \leq n$ aren't. When convenient, I will abbreviate the list $p_1; p_2; \ldots$ as *pp*. No change thereby occurs.

Now, consider the following principle:

(C) $\forall p \forall q (p; q <^* p \wedge q)$

where p and q may be atomic or complex propositions. The basic idea behind C is that, in some sense, the conjunction $\varphi \wedge \psi$ is derivative of the conjuncts. That is,

2. The extension to the case where *pp* is a countably infinite plurality is essentially the same: *pp* taken together are the minimal sufficient reason for q *iff* p_1, p_2,...p_n...taken together are a sufficient reason for q and p_1, p_2,...p_n...- p_m for $m \in \mathbb{N}$ aren't.

it's explained by them, and not vice versa. The conjunction $\varphi \wedge \psi$ is true *iff* both φ and ψ are true, but intuitively, it seems like the explanation runs right-to-left. In standard Boolean logic, one doesn't define the truth of conjuncts in terms of the truth of the conjunction. One instead first assigns truth values to atoms, and then assigns truth values to complex sentences *in terms of* the truth values of the atoms. This principle, then, is arguably a consequence of natural assumptions of Boolean logic carried over to propositions.

This introduction will require new versions of (S), (D), and (F) to fit our new connective. But this does not present insurmountable challenges. One simply adds the relevant modifications to the antecedent of $<^*$ along with quantifiers binding those singular propositional variables:

(S*) $\forall pp \forall q(pp <^* q \rightarrow \Box(\bigwedge pp \rightarrow q))$

(D*) $\forall p \forall qq[(p <^* \bigwedge qq) \rightarrow \forall q(q \prec qq \rightarrow p <^* q)]$

(F*) $\forall p \forall q[p <^* q \rightarrow (p \wedge q)]$

We call the theory gotten by supplementing TSR with C, as well as with the modified versions of (S), (D), and (F) TSR$^+$. Finally, we need the following for this proof:

(pp) If p is a member of the plurality pp, then $\vdash_{BL} p \prec pp$

Its intuitive appeal is fairly evident. It seems an obvious (perhaps analytic) truth about pluralities and their members that we should wish our derivation system to capture. With this in place, we can now prove

THEOREM 2. $\texttt{TSR}^+$ is $\texttt{BL}^+$-inconsistent

PROOF. $\texttt{TSR}^+ \vdash_{\texttt{BL}^+} (\texttt{D}^*)$, since $(\texttt{D}^*) \in \texttt{TSR}^+$. Further, $\{\forall p \forall qq[(p <^* \bigwedge qq) \rightarrow \forall q(q \prec qq \rightarrow p <^* q)]\} \rightarrow [(s <^* \bigwedge ss) \rightarrow (s \prec ss \rightarrow s <^* s)]$ is an instance of $(\texttt{A2})$, where ss is the plurality whose members are just multiple instances of s. So by $(\texttt{R2})$ and $(\texttt{D}^*)$ we have $\texttt{TSR}^+ \vdash_{\texttt{BL}^+} (s <^* \bigwedge ss) \rightarrow (s \prec ss \rightarrow r <^* s)$. Noting that $\bigwedge ss$ is just $s \wedge s$, we substitute uniformly and get that $\texttt{TSR}^+ \vdash_{\texttt{BL}^+} [s <^* (s \wedge s)] \rightarrow [(s \prec ss) \rightarrow (s <^* s)]$. $s <^* (s \wedge s)$ follows from $(\texttt{C}^*)$ and $(\texttt{A2})$ by $(\texttt{R1})$, so $\texttt{TSR}^+ \vdash_{\texttt{BL}^+} s <^* (s \wedge s)$. Since this is true, it follows that $\texttt{TSR}^+ \vdash_{\texttt{BL}^+} (s \prec ss) \rightarrow (s <^* s)$, by application of $(\texttt{R1})$ on $s <^* (s \wedge s)$. Since s is among the ss, an application of $(\texttt{R1})$ and $(\texttt{pp})$ gets us that $\texttt{TSR}^+ \vdash_{\texttt{BL}^+} s <^* s$. But note too that applying $(\texttt{R1})$ to the relevant instances of $(\texttt{Ir})$ and $(\texttt{A2})$ respectively get us that $\texttt{TSR}^+ \vdash_{\texttt{BL}^+} s \not<^* s$. So $\texttt{TSR}^+ \vdash_{\texttt{BL}^+} s \not<^* s$ and $\texttt{TSR}^+ \vdash_{\texttt{BL}^+} s <^* s$, as we wished to show. □

This doesn't indicate a conflict between $(\texttt{C})$ and $(\texttt{Ir})$. All those two would let us infer is that $\texttt{TSR}^+ \vdash_{\texttt{BL}^+} s <^* (s \wedge s)$, which doesn't get us the desired contradiction. Rather it's the interplay between $(\texttt{C})$, $(\texttt{Ir})$, and $(\texttt{D}^*)$ which generate the problem. So we must give up one of them. But which?

Section 3. $\texttt{TSR}^{**}$ is inconsistent

Presumably the defender of Hall's main argument will not want to give up $(\texttt{D}^*)$ or $(\texttt{Ir})$, since they're essential parts of the modal collapse argument. But $(\texttt{C})$ likewise seems hard to deny. It certainly *seems* true that propositional conjuncts are the sufficient reason for their conjunction. So perhaps what's needed is a restriction of (C) which disallows this argument. We might weaken it, for instance, to the following:

$$(\texttt{C}^*)\ \forall p \forall q[p \neq q \rightarrow (p; q <^* p \wedge q)]$$

Call the language which we obtain by adding a binary identity connective which joins singular and plural propositional variables and constants (but not the one to the other, in either respect) $\mathcal{L}^{=}_{PSR}$, and call the theory which results from adding ($\texttt{C}^*$) to TSR $\texttt{TSR}^*$. This theory is no longer inconsistent, since the proof for Theorem 2 fails to go through (we no longer have it that $\texttt{TSR}^+ \vdash_{\texttt{BL}^+} s <^* (s \wedge s)$).

Now consider the following principle:

$$(\texttt{Ir}^*)\ \forall p \forall q_1 \ldots \forall q_n \neg[(p = q_1 \vee \ldots p = q_n) \wedge (q_1 \ldots ; q_n <^* p)]$$

In words, ($\texttt{Ir}^*$) says that no proposition p is among the propositions which together constitute the minimal sufficient reason for p. This too is a desirable principle. Intuitively, we want to rule out such partial explanatory circles. To deny ($\texttt{IR}^*$) would be to assert that there is some proposition which helps explain itself. This seems like an undesirable result, since the sufficient reason for p should in an important sense be *prior* to p (be that temporally, ontologically, logically, etc), and hence all constituents of the minimal sufficient reason for p should be prior as well. And if p may be among the propositions which constitute a minimal sufficient reason for p, then p must in the relevant sense be prior to itself.

We need one final rule of inference to add to our theory, one which governs =:

(=) $\neg(p \leftrightarrow q)$ / $(p \neq q)$

The motivation for this rule is twofold. First, since = is not (necessarily) definable in terms of the primitive connectives, we would like some way of introducing propositions which contain it. Second, this rule seems obviously to be necessarily truth-preserving (at least classically, and Hall's theory is classical). For suppose that $\neg(p \leftrightarrow q)$ is true. Then p and q are not true together in some model.[3] Since this is so, then p and q cannot be identical, as identical propositions are true in all the same models. Hence, $(p \neq q)$ is true in all the models in which $\neg(p \leftrightarrow q)$ is first true.[4]

Let's write $\texttt{BL}^=$ for the logic gotten by adding (=) to $\texttt{B}^+$, $\texttt{TSR}^{**}$ for the theory gotten from adding $(\texttt{Ir}^*)$ to $\texttt{TSR}^*$ and taking its closure under $\texttt{BL}^=$. The trouble is that we now have the following

THEOREM 3. $\texttt{TSR}^{**}$ is $\texttt{BL}^=$-inconsistent.

PROOF. We saw above that

(1) $\texttt{TSR}^{**} \vdash_{\texttt{BL}^=} (\texttt{D})$.

We may obtain the formula $(p; q <^* p \wedge q) \rightarrow [(r = p \vee r = q) \rightarrow p; q <^* r]$ from (D) and (A2) by an application of (R1), so

3. See Hall 2021, Appendix 2 for the semantics of his theory.
4. The converse does not hold. Take p to be $s \rightarrow s$ and q to be $r \rightarrow (s \rightarrow r)$. Then clearly $p \neq q$ is true, but $\neg(p \leftrightarrow q)$ is not.

(2) $\mathtt{TSR}^{**} \vdash_{\mathsf{BL=}} (p; q <^* p \wedge q) \rightarrow [(r = p \vee r = q) \rightarrow p; q <^* r]$.[5]

Further, we can obtain the formula $p \neq q \rightarrow (p; q <^* p \wedge q)$ from ($\mathtt{C}^*$) and ($\mathtt{A2}$) by an application of ($\mathtt{R1}$), so

(3) $\mathtt{TSR}^{**} \vdash_{\mathsf{BL=}} p \neq q \rightarrow (p; q <^* p \wedge q)$.

Noting that $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ is a tautology of propositional logic, by ($\mathtt{A1}$) we have that

(4) $\mathtt{TSR}^{**} \vdash_{\mathsf{BL=}} \{[p \neq q \rightarrow (p; q <^* p \wedge q)] \wedge [p; q <^* p \wedge q \rightarrow (p \wedge q \wedge q)]\} \rightarrow [p \neq q \rightarrow (p \wedge q \wedge q)]$.

Note further that we can obtain the following from ($\mathtt{A2}$) and the modified version of ($\mathtt{F}$) by an application of ($\mathtt{R1}$):

(5) $\mathtt{TSR}^{**} \vdash_{\mathsf{BL=}} p; q <^* p \wedge q \rightarrow (p \wedge q \wedge q)$

And now, since we have (3) and (5), we may conclude that:

(6) $\mathtt{TSR}^{**} \vdash_{\mathsf{BL=}} [p \neq q \rightarrow (p; q <^* p \wedge q)] \wedge [p; q <^* p \wedge q \rightarrow (p \wedge q \wedge q)]$[6]

---

5. Again, technically I am eliding a step, as there is no principle that lets me uniformly substitute some disjunction of identities whenever I see $p \prec pp$. But, again, I think this is unproblematic, as it seems obviously true that if p is among pp then p is identical to one of the (non-set-theoretic) members of pp.

6. Again I am making an assumption that is not explicitly stated in Hall's text, but I don't think that this is problematic either. If a theory which extends basic classical quantified logic proves both conjuncts, surely it proves their conjunction.

Now since we have the formula in (6), we use (R1) on it and on the formula in (4) to obtain that:

(7) $\texttt{TSR}^{**} \vdash_{\texttt{BL=}} p \neq q \to (p \wedge q \wedge q)$

Since $(p \wedge q \wedge q) \to (p \wedge q)$ is a tautology of propositional logic, we have by (A1) that

(8) $\texttt{TSR}^{**} \vdash_{\texttt{BL=}} (p \wedge q \wedge q) \to (p \wedge q)$

And since $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is likewise a tautology of propositional logic, we have by (A1) that

(9) $\texttt{TSR}^{**} \vdash_{\texttt{BL=}} [(p \neq q \to (p \wedge q \wedge q)) \wedge ((p \wedge q \wedge q) \to (p \wedge q))] \to (p \neq q \to (p \wedge q))$

Since we have (7) and (8), we have the following:

(10) $\texttt{TSR}^{**} \vdash_{\texttt{BL=}} [p \neq q \to (p \wedge q \wedge q)] \wedge [(p \wedge q \wedge q) \to (p \wedge q)]$[7]

Now we obtain, by using (R1) on the formulae in (9) and (2), that

(11) $\texttt{TSR}^{**} \vdash_{\texttt{BL=}} p \neq q \to (p \wedge q)$

Since $(p \wedge q) \to p$ and $[(p \to q) \wedge (q \to r)] \to (q \to r)$ are tautologies of propositional logic, by (A1), the formulae in (10) and (11), and repeated applications of

7. Again I make the same assumption that I did to get to (6), and again I think it's unproblematic.

(R1) we have that:

(12) $\texttt{TSR}^{**} \vdash_{\texttt{BL=}} p \neq q \rightarrow p.$

Note that, since we obtained the instances of p and q by (A2), the choice of p and q was arbitrary. Suppose then that p is $r \vee \neg r$, and q is $\neg p$. Note that p is a propositional tautology, so q is a contradiction. So by (A1), we have

(13) $\texttt{TSR}^{**} \vdash_{\texttt{BL=}} \neg(p \leftrightarrow q)$

since the negation of a contradiction, which under these definitions $p \leftrightarrow q$ is, is a tautology. Using (=) on (14) yields that

(14) $\texttt{TSR}^{**} \vdash_{\texttt{BL=}} p \neq q$

Using (R1) on (14) and (12), we obtain that

(15) $\texttt{TSR}^{**} \vdash_{\texttt{BL=}} p$

which is no surprise. But since p and q were arbitrary, if we let q be $r \vee \neg r$ and p be $\neg q$ instead, we also have

(16) $\texttt{TSR}^{**} \vdash_{\texttt{BL=}} \neg q$

Which, substituting for p and q appropriately in each successive stage, gives us

(17) $\texttt{TSR}^{**} \vdash_{\mathtt{BL=}} r \vee \neg r$

and

(18) $\texttt{TSR}^{**} \vdash_{\mathtt{BL=}} \neg(r \vee \neg r)$

So $\texttt{TSR}^{**}$ is inconsistent, as we wished to show.

□

Section 4. What to give up

The proponent of the modal collapse argument intends to make the adherent of the PSR renounce (or modify) it by showing that it leads to unpalatable results. Likewise, I aim to make the defender of Hall's modal collapse argument give up one of their premises by showing that they lead to an inconsistency. But which should they give up?

Subsection 4.1. $\texttt{TSR}^{+}$. Recall that $\texttt{TSR}^{+}$ is $\texttt{TSR}$ with a supplementation in the language and principles. So, in order to evade inconsistency, a proponent of $\texttt{TSR}^{+}$ must either reject the supplemented language or reject the supplementary principle, ($\texttt{C}$). Rejecting the supplemented language, $\mathcal{L}^{+}_{PSR}$, seems to be difficult to do. Either one would do so for purely ad hoc reasons ("it defuses the argument"), or one would have to show something amiss with the concept that $<^{*}$ is intended to regiment.

Perhaps one may do so. Perhaps there is no *unique* minimal sufficient reason for a proposition, but instead several. Does that afford the defender of this argument the escape they need?

I don't think so. Nothing in the argument relies on there being a *unique* minimal sufficient reason for some proposition, only that there be *some* minimal sufficient reason. We would then reformulate the clause for $<^*$ by simply dropping condition (iii) and allowing other pluralities to satisfy (i) and (ii).

Denying the supplementary principle ($\texttt{C}^*$) seems the most plausible way out of the argument. For, when coupled with ($\texttt{F}^*$), it entails that arbitrary conjunctions exist.[8]. In other words, we have the following

THEOREM 4. $\texttt{TSR}^+ \vdash_{\texttt{BL}^+} p \wedge q$ for any p and q.

PROOF. By an application of ($\texttt{R1}$) on an instance of ($\texttt{A2}$) and ($\texttt{C}$), we have that

(1) $\texttt{TSR}^+ \vdash_{\texttt{BL}^+} p; q <^* p \wedge q$

Further, by an application of ($\texttt{R1}$) on an instance of ($\texttt{A1}$) and ($\texttt{F}$), we have that

(2) $\texttt{TSR}^+ \vdash_{\texttt{BL}^+} p; q <^* (p \wedge q) \rightarrow [(p \wedge q) \wedge (p \wedge q)]$

And by an application of ($\texttt{R1}$) on the formulae in (1) and (2), we have that

(3) $\texttt{TSR}^+ \vdash_{\texttt{BL}^+} (p \wedge q) \wedge (p \wedge q)$

Finally, noting that $[(p \wedge q) \wedge (p \wedge q)] \rightarrow (p \wedge q)$ is an instance of a propositional tautology, we have, by an application of ($\texttt{A1}$) on an instance of ($\texttt{A1}$) and the formula in (3), that

8. Thanks to [**redacted**] for pointing this out to me.

(4) $\texttt{TSR}^+ \vdash_{\texttt{BL}^+} p \wedge q$

just as we wished to show. □

Even worse, we have the following

COROLLARY 1. $\texttt{TSR}^+$ is inconsistent.

PROOF. Almost immediate from THEOREM 3. Since $\texttt{TSR}^+ \vdash_{\texttt{BL}^+} p \wedge q$, and $(p \wedge q) \rightarrow p$ is a tautology of propositional logic, we have that $\texttt{TSR}^+ \vdash_{\texttt{BL}^+} p$ for any p whatever by (R1) on the result of THEOREM 3 and (A1). In particular this means that $\texttt{TSR}^+ \vdash_{\texttt{BL}^+} r$ and $\texttt{TSR}^+ \vdash_{\texttt{BL}^+} \neg r$ for some (indeed any) proposition r. So, $\texttt{TSR}^+$ is $\texttt{BL}^+$-inconsistent. □

This motivates a rejection of ($\texttt{C}^*$). Or at least it is meant to. Since this result is generated by both (C) and (F), what it really motivates is a rejection of one or the other of these principles. But I think that there are considerations that tell against accepting ($\texttt{F}^*$) in the first place. Take for instance the sentence

(*) That there are finitely many primes is the sufficient reason that Euclid's theorem is false.

It seems to me perfectly reasonable to say that this sentence is true *as stated*, even though its translation under ($\texttt{F}^*$) it must be false. So much the worse for ($\texttt{F}^*$), says I. It seems both perfectly intelligible and actually true for two propositions to stand in the sufficient reason relation without either actually existing. Perhaps, one might say, facts about the sufficient reason relation between propositions may obtain without the embedded propositions existing. By analogy, the disjunctive proposition $p \vee \neg p$ is both true and exists, but since p exists, $\neg p$ can't (or vice versa). So a proposition may exist or be true without at least one of its component propositions existing.

Indeed, we may have the extreme case where neither embedded proposition is true but the larger one is true (or exists). Consider the complex proposition $(p \wedge \neg p) \rightarrow (p \wedge \neg p)$. This is true – indeed, necessarily true. But both the antecedent and the consequent are false. So we may have a true (or existent) complex proposition where the embedded propositions are not true (or do not exist).

This is a proof of concept. Any further objection which says that factivity is required for sufficient reason, then, must turn on the specific meaning of sufficient reason rather than general concerns about true (or existent) propositions being composed of false (or non-existent) ones. The issue that anyone who rejects ($\mathtt{F}^*$) must confront, instead, is how to secure that a contingent proposition cannot have as its sufficient reason a necessary proposition. This is because it follows pretty quickly in $\mathtt{TSR}^+$ (because of ($\mathtt{S}^*$)) that if a proposition is necessary, any proposition which it is the sufficient reason of must also be necessary.[9] But this differs from the rejection of necessitarianism generated by avoiding modal collapse.

At this point, then, I think one may plausibly assume that ($\mathtt{F}^*$) is false *independently* of Theorems 2 and 3. This of course doesn't show that ($\mathtt{C}^*$) is true, but since we have independent reasons for thinking ($\mathtt{C}$) is true, it motivates choosing ($\mathtt{C}$) over ($\mathtt{F}$).

Subsection 4.2. $\mathtt{TSR}^=$. Still, perhaps the defender of modal collapse will reject ($\mathtt{C}$) anyway. But, recall, we have shown what happens even when one adopts a deflated version of it, ($\mathtt{C}^*$). Further, we have the following adaptation of Theorem

9. One might fix this by adopting a polymodal version of $\mathtt{TSR}^+$, where the modality in ($\mathtt{A4}$) differs from that in ($\mathtt{I}^*$) and ($\mathtt{S}^*$).

4:

THEOREM 5. $\texttt{TSR}^{=} \vdash_{\texttt{BL}^{=}} p \wedge q$ for any distinct p and q.

PROOF. By an application of (R1) on an instance of (A2) and (C*), we have that

(1) $\texttt{TSR}^{=} \vdash_{\texttt{BL}^{=}} (p \neq q) \rightarrow (p; q <^{*} p \wedge q)$

Since $\{[(p \neq q) \rightarrow (p; q <^{*} p \wedge q)] \wedge [(p; q <^{*} p \wedge q) \rightarrow (p \wedge q)]\} \rightarrow [(p \neq q) \rightarrow (p \wedge q)]$ is an instance of a propositional tautology, we have

(2) $\texttt{TSR}^{=} \vdash_{\texttt{BL}^{=}} \{[(p \neq q) \rightarrow (p; q <^{*} p \wedge q)] \wedge [(p; q <^{*} p \wedge q) \rightarrow (p \wedge q)]\} \rightarrow [(p \neq q) \rightarrow (p \wedge q)]$

By an application of (R1) on an instance of (A2) and (F), we have that

(3) $\texttt{TSR}^{=} \vdash_{\texttt{BL}^{=}} (p; q <^{*} p \wedge q) \rightarrow (p \wedge q)$

And since $\texttt{BL}^{=}$ is an extension of classical propositional logic, we have that

(4) $\texttt{TSR}^{=} \vdash_{\texttt{BL}^{=}} [(p \neq q) \rightarrow (p; q <^{*} p \wedge q)] \wedge [(p; q <^{*} p \wedge q) \rightarrow (p \wedge q)]$

So now, by an application of (R1) on the formulae in (2) and (4), we have that

(5) $\texttt{TSR}^{=} \vdash_{\texttt{BL}^{=}} (p \neq q) \rightarrow (p \wedge q)$

So $\mathit{TSR}^{=} \vdash_{BL^{=}} p \wedge q$ for any distinct p and q, as we wished to show.

□

And then, of course, we have the following

COROLLARY 2. $\mathtt{TSR}^{+}$ is inconsistent.

PROOF. This is proved exactly how we proved Corollary 1, except from THEOREM 5.

□

So again, we have a conflict between ($\mathtt{F}^*$) and ($\mathtt{C}^*$). So to hold onto the argument, it seems, the defender of modal collapse will need to reject any conjunctive principle. This seems a heavy price to pay.

Section 5. Revising (F)

Or is it? Perhaps we can revise ($\mathtt{F}^*$) in a way that allows us to keep some conjunctive principle while still allowing the defender of modal collapse to retain their argument. For, recall, ($\mathtt{F}^*$) played an essential part in generating the modal collapse argument.

The problem with (F) was that it seemed to violate intuitions about the relationship between propositions which are false (or that don't exist). But perhaps the defender of modal collapse can accept this, and instead put forward a revised notion of the sufficient reason relation, an essentially *factive* one. In other words, we add to $\mathcal{L}^{=}_{PSR}$ the following defined connective:

- An (n+1)-ary connective $<^F$ for every $n \in \mathbb{N}$ to formalize "φ_1, φ_2,...φ_n are together the minimal factive sufficient reason for ψ," written as $\varphi_1; \varphi_2 \ldots; \varphi_n <^F \psi$, and given the definition $\varphi_1; \varphi_2 \ldots; \varphi_n <^F \psi := (\varphi_1; \varphi_2 \ldots; \varphi_n <^* \psi) \wedge (\bigwedge \varphi_i \wedge \psi)$[10]

In other words, the φs taken together are the minimal factive sufficient reason for ψ just in case they're the minimal sufficient reason for ψ and all of them are true (or exist). The resulting language is no more expressive than $\mathcal{L}^{=}_{PSR}$, since the new connective is defined in terms of ones already used in $\mathcal{L}^{=}_{PSR}$. We then drop (F) entirely from all versions of (TSR) and replace all instances of $<^*$ with $<^F$, like so:

($\mathtt{I}^F$) $\forall pp[\Box \bigwedge pp \to \forall p(p \prec pp \to \Box p)]$

(T) $\forall pp[\forall p(p \prec pp \to p) \to \bigwedge pp]$

($\mathtt{S}^F$) $\forall p \forall q(p <^F q \to \Box(p \to q))$

($\mathtt{Ir}^F$) $\forall p(p \not<^F p)$

($\mathtt{D}^F$) $\forall p \forall qq[(p <^F \bigwedge qq) \to \forall q(q \prec qq \to p <^F q)]$

($\mathtt{PSR}^F$) $\forall p[Cp \to \exists q(q <^F p)]$

Call the resulting theory $\mathtt{TSR}^F$. A defender of modal collapse might then think to give a version of Hall's argument that is almost identical to the original, except that all instances of $<^*$ are replaced with $<^F$. This may be true. But then it is incumbent upon such a defender to produce such a proof.

Notice that now ($\mathtt{Ir}^F$) becomes a bit odd. It could be true with a reflexive sufficient reason loop, provided that one of the two propositions at least was not true (did not exist). This seems like a failure to capture what ($\mathtt{Ir}^F$) was supposed to

10. Again, thanks to [**redacted**] for suggesting this to me.

capture. Still, one could simply retreat back to ($\mathtt{Ir}^*$) and give up the modification. That should work just as well.

Section 6. Reconsidering (D)

But darker stormclouds loom. If we write out ($\mathtt{D}^F$) in terms only of $<^*$, we get the following:

$$(\mathtt{D}^F)\ \forall p \forall qq \big([(p <^* \bigwedge qq) \wedge (\bigwedge qq \wedge p)] \rightarrow \forall q \{ q \prec qq \rightarrow [(p <^* q) \wedge (q \wedge p)] \} \big)$$

What's wrong here? The issue is that this may be true for reasons having nothing to do with the relationship of sufficient reason. Suppose, for instance, that $\bigwedge qq$ and p are both true (or exist), but that p is not the minimal sufficient reason for $\bigwedge q$. Then ($\mathtt{D}^F$) is true. What I mean to say here is that this is not a principle whose truth turns on what it means to be a sufficient reason. To see this again, suppose instead that $<^*$ were reversed. In other words, suppose that the proposition(s) on the right hand side were the sufficient reason for the proposition(s) on the left hand side. Then the conditional may still be true under the circumstances described above. In fact, suppose we had a class of models where *no* relation of minimal sufficient reason obtained between any of the propositions in any domain. Call this a Cthulhu-class: In these models, just *happen*, without any connection, rhyme, or reason. Then ($\mathtt{D}^F$) would be true *in every world in every domain of every model in that class*, since the antecedent of that conditional is false.

This is a problem, and it also infects the original, unmodified (D). We would like it to say something about the relation of sufficient reason – that is, to characterize it – and hence be false when there is no such relation. But as we have seen, in every model in which there is no such relation it is *true*. This makes it informative, perhaps, but not informative about the relation of *sufficient reason*. If it tells us anything about sufficient reasons, it seems like it only does so accidentally.

It may also be false in a wide range of worlds for reasons having nothing to do with the relation of sufficient reason, or the PSR. Consider the class of worlds which fulfill the following conditions:

- There are countably infinitely many propositions ...$p_{-2}, p_{-1}, p_0, p_1, p_2$,...which are true (exist) at w.
- For each proposition p_n, $n \in \mathbb{Z}$, $p_{n-1} <^* p_n$

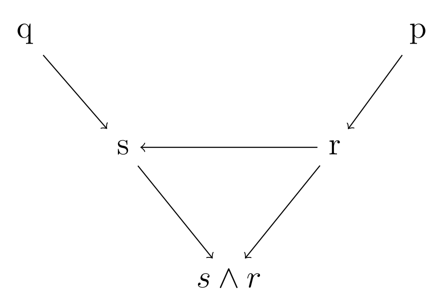
Note that, for arbitrary conjunctions of these p_i, the conjunction is explained by its conjuncts, per ($\texttt{C}^*$). Call worlds like this Hume-worlds. In these worlds, ($\texttt{PSR}^F$) is true: every proposition at these worlds is explained by some minimal sufficient reason, even conjunctions over all the propositions per ($\texttt{C}^F$). So every proposition has a sufficient reason. The antecedent of ($\texttt{D}^F$), then, is satisfied: the conjunction $\bigwedge qq$ has a minimal sufficient reason, and both the conjunction and that sufficient reason exist (since the minimal sufficient reason is just all the conjuncts). But the consequent is *false*: If q is among the conjuncts of $\bigwedge$, then by ($\texttt{Ir}^*$) p cannot figure in its own minimal sufficient reason, and hence the conjunction in the consequent is false.

So it seems that there are issues with ($\texttt{D}$): In many cases it seems to be true or false in ways that don't tell us much about the relation of sufficient reason. But by itself, this may not be as important an objection. Thankfully, however, that doesn't end the trouble with ($\texttt{D}$). We also have potential counterexamples.[11] Consider for

11. The following sort of example emerged from a discussion with [**redacted**], whom I would like to thank. Any errors or infelicities in the counterexample are, of course, his fault and his fault alone.

instance p, the proposition which expresses that I have a gene that determines my melanin content, and the proposition q which expresses the fact that all the necessary and sufficient conditions for someone who has that melanin content to get sunburnt in a particular way are met.[12] And consider too the proposition s which expresses that I got sunburnt, and the proposition r which expresses my melanin content. Then clearly $p, q <^* r \wedge s$. But note that $p, q <^* r$ is false: this is *not* the minimal sufficient reason for me having that melanin content. That's just p. But if some suitably modified version of (D) is true, then $p, q <^* s$ would be true. So – I conclude – that suitably modified version of (D) is false.

The general structure of the counterexample is this. Suppose that r has p for a minimal sufficient reason, and s has r and q for a minimal sufficient reason. Then in general we will have $p; q <^* r \wedge s$, but not $p; q <^* r$. This can be represented by the following graph:



The conditions on this graph are as follows:

- If q lies within the transitive closure of p, then p is part of the minimal sufficient reason for q.

12. I do not here assume that genes *do determine* one's melanin content, only that they *may*. If you like, insert a proposition expressing that all the necessary and sufficient conditions for my having that phenotype, be they genetic, environmental, or otherwise, are met.

- p_1, ...p_n are the minimal sufficient reason for q iff you can reach q from each p_m by a directed walk and there are no other nodes that you can reach q by a directed walk from which cross q_m.

Thus in this graph, p and q together are the minimal sufficient reason for $s \wedge r$, since the node for $s \wedge r$ lies within the transitive closure of each. But together they are not for r, since r doesn't lie within the transitive closure of q.

Notice too that even if we relax the second condition and allow for directed walks which *do* pass members of the minimal sufficient reason, we will still have a problem. According to this graph, we would have $p; q; r; s <^F r \wedge s$. But of course $p; q; r; s <^F r$ is false, because of ($\texttt{Ir}^F$). Further, this counterexample is not generated using (C) – in fact, it is one in which (C) is false, since s and r are not together the minimal sufficient reason for $s \wedge r$. Because of this, the defender of (D) or its progeny can't reply by denying (C). While I am committed to (C)'s actual truth, I don't need to be so committed in general to generate this counterexample. It works given just the premises that the defender of Hall's argument would accept.[13]

So we can use this structure to generate a potentially limitless number of counterexamples. Suppose, for instance, p expresses the proposition that a mad scientist turned on an electrode in my brain that determines me to look for a sandwich, r the proposition that I looked for a sandwich, q the proposition that there was a sandwich in the refrigerator, and s the proposition that I ate the sandwich. Then while the propositions that the mad scientist turned on the electrode and that there was a sandwich in the refrigerator taken together are a minimal sufficient reason for

13. We might alternately define the notion of an *immediate* minimal sufficient reason, in which only those propositions which immediately provide the minimal sufficient reason for some proposition are involved. This would pair up with the notion of an *extended* minimal sufficient reason, in which all propositions lying within the inverse transitive closure of some proposition's graph (as above) enter into its sufficient reason.

the conjunctive proposition that I both looked for and ate a sandwich, they aren't a minimal sufficient reason for the proposition that I looked for a sandwich. Whether or not this or the sunburn example are *actual* counterexamples, in the sense of actually obtaining, doesn't matter much. All that matters is that whenever we have an explanatory structure like that of the graph, $(\mathtt{D}^*)$ is false. A

Things get worse. There is of course a counterpart of THEOREM 2 for $(\mathtt{D}^F)$, namely that

THEOREM 6. $(\mathtt{D}^F)$, $(\mathtt{C}^F)$, $(\mathtt{Ir}^F) \vdash_{\mathtt{BL}} \bot$

PROOF. Suppose $(\mathtt{D}^F)$, $(\mathtt{C}^F)$, $(\mathtt{Ir}^F) \vdash_{BL} p$ and $(\mathtt{D}^F)$, $(\mathtt{C}^F)$, $(\mathtt{Ir}^F) \vdash_{BL} q$ for $p \neq q$. In fact take p to be $(\mathtt{D}^F)$ and q to be $(\mathtt{C}^F)$. Then $(\mathtt{D}^F)$, $(\mathtt{C}^F)$, $(\mathtt{Ir}^F) \vdash_{BL} p \wedge q$, and $(\mathtt{D}^F)$, $(\mathtt{C}^F)$, $(\mathtt{Ir}^F) \vdash_{BL} p, q <^F p \wedge q$. Then it follows, using $(\mathtt{pp})$, that $(\mathtt{D}^F)$, $(\mathtt{C}^F)$, $(\mathtt{Ir}^F) \vdash_{BL} p \prec qq$, where qq is just the plurality consisting of p and q. And from that it follows that $(\mathtt{D}^F)$, $(\mathtt{C}^F)$, $(\mathtt{Ir}^F) \vdash_{BL} p, q <^F p$. But by $(\mathtt{Ir}^F)$, it follows that $(\mathtt{D}^F)$, $(\mathtt{C}^F)$, $(\mathtt{Ir}^F) \vdash_{BL} p, q \not<^F p$. So $(\mathtt{D}^F)$, $(\mathtt{C}^F)$, $(\mathtt{Ir}^F) \vdash_{BL} \bot$, as we wished to show. □

So $(\mathtt{D}^F)$, $(\mathtt{C}^F)$, and $(\mathtt{Ir}^F)$ are inconsistent . Which do we give up? Since I have given independent arguments for $(\mathtt{C}^F)$ and $(\mathtt{Ir}^F)$, and multiple independent arguments *against* $(\mathtt{D}^F)$, my suggestion that we should give up $(\mathtt{D}^F)$. Lay it to rest.

Concluding Remarks

I've argued in this paper that the assumptions of Hall's arguments (and hence of any modal collapse argument which uses similar ones) generate an inconsistency when combined with another extremely plausible principle. Further, certain of the original principles, while initially plausible, suffer from potentially fatal problems. This should tell us that there is something wrong with them.

Now this obviously doesn't show that PSR is *true*. It may just be that certain propositions fail to have any sufficient reason whatever. Perhaps, for example, some indeterministic interpretation of quantum mechanics is correct, and (assuming the PSR requires determinism) hence the PSR is false. But what I hope to have shown is that it is difficult to maintain that it is false because of modal collapse arguments like Hall's. Necessitarianism is lost, and contingency regained.

References

Amijee, Fatema. 2020. "Principle of Sufficient Reason." In Raven 2020, chap. 4.

Bennett, Jonathan. 1984. *A Study of Spinoza's Ethics.* Indianapolis, Indiana: Hackett Publishing Company.

Della Rocca, Michael. 2010. "PSR." *Philosophers' Imprint* 10 (7): 1–13.

Hall, Geoffrey. 2021. "Indefinite extensibility and the principle of sufficient reasons." *Philosophical Studies* 168:471–492.

Levey, Samuel. 2016. "The Paradox of Sufficient Reason." *Philosophical Review,* no. 3, 397–430.

McDaniel, Kris. 2019. "The principle of sufficient reasona and necessitarianism." *Analysis* 79 (2): 230–236.

Pruss, Alexander. 2006. *The Principle of Sufficient Reason: A Reassessment.* New York: Cambridge University Press.

Raven, Michael J., ed. 2020. *The Routledge Handbook of Metaphysical Grounding.* New York: Routledge.

Tomaszewski, Christopher M. P. 2016. “The Principle of Sufficient Reason Defended: There Is No Conjunction of All Contingently True Propositions.” *Philosophia* 44:267–274.

van Inwagen, Peter. 1986. *An Essay on Free Will.* Oxford: Clarendon Press.